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Unsteady MHD convective flow within a parallel plate rotating channel with thermal source/sink in a porous medium under slip boundary conditions

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Abstract

Unsteady hydromagnetic convective flow of a viscous incompressible electrically conducting heat generating/absorbing fluid within a parallel plate rotating channel in a uniform porous medium under slip boundary conditions is investigated. Exact solution of the governing equations for fully developed flow is obtained in closed form. Expressions for skin friction due to primary and secondary flows and Nusselt number at the plate $\eta = 1$ are also derived. Asymptotic behavior of the solution for the fluid velocity is analyzed for large values of frequency parameter ω to gain some physical insight into the flow pattern. The numerical values of the primary and secondary velocities and fluid temperature are displayed graphically versus channel width variable η for various values of pertinent flow parameters whereas numerical values of skin frictions due to primary and secondary flows and Nusselt number at the plate $\eta = 1$ are presented in tabular form for different values of pertinent flow parameters.

Keywords: Thermal source/sink, slip boundary conditions, periodic pressure gradient, convective flow, magnetic field, rotation, porous medium.

1. Introduction

Flow of a viscous fluid in a rotating medium is of considerable importance due to the occurrence of various natural phenomena and for its application in various technological situations which are governed by the action of Coriolis force. The broad subjects of oceanography, meteorology, atmospheric science and limnology all contain some important and essential features of rotating fluids. The viscous fluid flow problems in rotating medium under different conditions and configurations are investigated by many researchers in the past to analyze various aspects of the problem. Mention may be made of the research studies of Greenspan and Howard (1963), Holton (1965), Walin (1969), Siegman (1971), Puri (1974), Puri and Kulshrestha (1974), Mazumder (1991), Ganapathy (1994), Hayat *et al* (2001), Hayat and Hutter (2004) and Das *et al.* (2008). The study of simultaneous effects of rotation and magnetic field on the fluid flow problems of a viscous incompressible electrically conducting fluid may find applications in the areas of geophysics, astrophysics and fluid engineering. An order of magnitude analysis shows that, in the basic field equations, the effects of Coriolis force are more significant as compared to that of inertial and viscous forces. Furthermore, it may be noted that Coriolis and magnetohydrodynamic forces are comparable in magnitude and Coriolis force induces secondary flow in the flow-field. Taking into consideration these facts Vidyanidhi (1969), Nanda and Mohanty (1971), Mazumder (1977), Jana *et al* (1977), Jana and Datta (1980), Prasad Rao *et al* (1982), Seth and Maiti (1982), Mandal *et al* (1982), Mandal and Mandal (1983), Raman Rao and Linga Raju (1990), Nagy and Demendy (1993, 1995), Ghosh and Bhattarchjee (2000), Seth and Singh (2008), Seth and Ansari (2009), Seth *et al* (2009) and Ghosh *et al* (2009) studied steady MHD flow of a viscous incompressible electrically conducting fluid in a rotating channel under different conditions considering various aspects of the problem. Investigation of oscillatory flow in a rotating channel is important from practical point of view because fluid oscillations may be expected in many MHD devices and natural phenomena where fluid flow is generated due to oscillating pressure gradient or due to vibrating walls. Keeping in view this fact Mukherjee and Debnath (1977), Seth and Jana (1980), Singh (2000), Ghosh (1993), Ghosh and Pop (2003), Hayat *et al* (2004) and Guria *et al* (2009) investigated oscillatory flow of a viscous incompressible electrically conducting fluid in a rotating channel under different conditions to analyze various aspects of the problem. Rahman and Sattar (1999) studied MHD free convection and mass transfer flow with oscillating plate velocity and constant heat source in a rotating frame of reference. In all these investigations "no-slip" boundary condition is considered for the velocity field. However, in some application e.g. in microfluidic and nanofluidic devices where the surface to volume ratio is large, the slip behavior is more typical and slip boundary condition is usually used for the velocity field (Darhuber and Troian, 2005) which was first proposed by Navier in the year 1823. There exist many physical reasons for the slip over hydrophobic surfaces among which are: molecular slip (Blake, 1990) and small dipole-moment of polar liquids (Melin *et al*, 2004). Also wall slip can occur in the working fluid which contains concentrated suspensions (Soltani and Tilmazer, 1998). Keeping in view these facts the effects of fluid slippage at the wall for Couette flow under steady state condition for gases are studied by Marques *et al* (2000) whereas Khaled and Vafai (2004) investigated Stokes and Couette flows produced by an oscillatory motion of a wall under slip boundary conditions. Soundalgekar (1970) considered hydromagnetic fluctuating flow past an infinite porous plate in slip flow regime while Sastry and Bhadram (1976) studied magnetogasdynamic flow past an infinite porous plate in slip flow regime. Makinde and Osalusi (2006) investigated MHD steady flow in a channel with permeable boundaries under slip boundary conditions. Linga Raju (2007) considered steady hydromagnetic flow in a rotating channel with non conducting walls in slip flow regime. Smolentsev (2009) investigated three types of MHD flow problems assuming hydrodynamic slip condition at the interface between the electrically conducting fluid and insulating walls which are: (i) Hartmann flow; (ii) fully developed flow in a rectangular duct and (iii) quasi two dimensional turbulent flow. Abelman *et al.* (2009a) considered steady MHD flow of a third grade fluid past a rigid plate with slip boundary condition in a rotating frame whereas Abelman *et al.* (2009b) studied steady MHD Couette flow of thermodynamic compatible third grade fluid filling the porous space in a rotating frame taking partial slip into account.

Unsteady convective flow of a viscous incompressible heat generating/absorbing fluid is of considerable importance due to appreciable temperature difference between the surface and ambient fluid in so many fluid flow problems of physical interest. Internal heat generation/absorption plays significant role in various physical phenomena such as fluids undergoing exothermic or endothermic chemical reactions (Vajravelu and Nayfeh, 1992), convection in Earth's mantle (McKenzie et al, 1974), application in the field of nuclear energy (Crepeau and Clarksean, 1997), post accident heat removal (Baker et al, 1976), fire and combustion modeling (Delichatsios, 1988) and the development of metal waste from spent nuclear fuel (Westphal, 1994). Although exact modeling of internal heat generation or absorption is quite difficult, some simple mathematical models yet idealized can express its average behavior for most physical situations. Sparrow and Cess (1961) considered temperature-dependent heat absorption in their investigation of steady stagnation point flow and heat transfer. Moalem (1976) studied steady state heat transfer in a porous medium with temperature-dependent heat generation. Jha and Ajibade (2009) considered free convection flow of heat generating/absorbing fluid between vertical porous channel due to periodic heating of the walls of the channel and temperature-dependent heat generation/absorption. Kamel (2001) investigated unsteady MHD convection flow through a porous medium bounded by an infinite vertical porous plate with temperature-dependent thermal source/sink. Chamkha (2004) considered unsteady two dimensional convective heat and mass transfer boundary layer flow of a viscous, incompressible, electrically conducting and heat absorbing fluid past a semi-infinite vertical permeable plate with temperature-dependent heat absorption.

The aim of the present paper is to study unsteady hydromagnetic convective flow of a viscous, incompressible, electrically conducting and heat generating/absorbing fluid within a parallel plate vertical channel in a uniform porous medium under hydrodynamic slip boundary conditions with temperature dependent thermal source/sink when both the fluid and channel are in a state of rigid body rotation with uniform angular velocity about an axis perpendicular to the planes of the plates. Fluid within the channel is permeated by a uniform transverse magnetic field applied in a direction which is parallel to the axis of rotation.

2. Formulation of the Problem and its Solution

Consider flow of a viscous, incompressible, electrically conducting and heat generating/absorbing fluid within a parallel plate vertical channel (i.e. $z = 0$ to $z = L$) in a uniform porous medium in the presence of a uniform transverse magnetic field B_0 applied in a direction which is parallel to z-axis about which both the fluid and channel are in a state of rigid body rotation with uniform angular velocity Ω. Plate $z = 0$ of the channel is kept at uniform temperature T_0 whereas plate $z = L$ of the channel is maintained at an oscillating temperature $T_0 + (T_w - T_0)\cos\omega' t'$. ω', T_w and t' are, respectively, frequency of oscillations, temperature of the plate $z = L$ in steady state (i.e. when $\omega' = 0$) and time. Flow within the channel is induced by a periodic pressure gradient $\partial p/\partial x = 2R\cos\omega' t'$ applied in x-direction, R being a constant. Physical model of the problem is presented in figure 1.

The equations of motion for a viscous, incompressible, electrically conducting and heat generating/absorbing fluid in a rotating medium are

$$\frac{\partial \vec{q}}{\partial t'} + (\vec{q}.\nabla)\vec{q} + 2\Omega\hat{k}\times\vec{q} = -\frac{1}{\rho}\nabla p' + \upsilon\nabla^2\vec{q} - \frac{\upsilon}{K'}\vec{q} + \frac{1}{\rho}\left(\vec{J}\times\vec{B}\right) + g\beta'(T' - T_0)\hat{i}\ , \tag{1}$$

$$\nabla.\vec{q} = 0\,, \tag{2}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t'}\,, \tag{3}$$

$$\nabla \times \vec{B} = \mu_e \vec{J}\,, \tag{4}$$

$$\nabla.\vec{B} = 0\,, \tag{5}$$

Ohm's law for a moving conductor is

$$\vec{J} = \sigma(\vec{E} + \vec{q} \times \vec{B})\,, \tag{6}$$

and energy equation for the problem is

$$\frac{\partial T'}{\partial t'} + (\vec{q}.\nabla)T' = \frac{k}{\rho C_p}\nabla^2 T' - \frac{Q_0}{\rho C_p}(T' - T_0)\,, \tag{7}$$

where $\vec{q}$, $\vec{B}$, $\vec{J}$, $\vec{E}$, T', ρ, υ, K', g, β', σ, μ_e, k, C_p, $\hat{i}$, $\hat{k}$, t', p' and Q_0 are, respectively, fluid velocity, magnetic field, current density, electric field, fluid temperature, fluid density, kinematic coefficient of viscosity, permeability of porous medium, acceleration due to gravity, volumetric coefficient of thermal expansion, electrical conductivity, magnetic permeability, thermal conductivity, specific heat at constant pressure, unit vector along x-axis, unit vector along z-axis, time, pressure including centrifugal force and dimensional heat generation/absorption coefficient. It may be noted that $Q_0 < 0$ for heat generation and $Q_0 > 0$ for heat absorption.

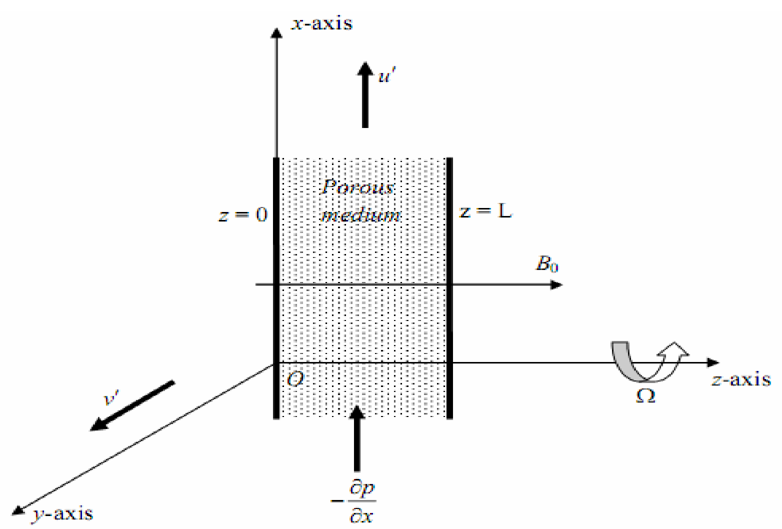

Figure 1. Physical model of the problem

Since plates of the channel are of infinite extent in x and y directions and electrically non-conducting and flow is fully developed so all physical quantities, except pressure p', depend on z and t' only i.e pressure p' is function of x, y, z and t' whereas fluid velocity $\vec{q}$, fluid temperature T', skin friction and Nusselt number are functions of z and t' only.

Taking into consideration assumptions made above fluid velocity $\vec{q}$, magnetic field $\vec{B}$, current density $\vec{J}$ and electric field $\vec{E}$ are given by

$$\vec{q} \equiv (u', v', 0)\,,\ \vec{B} \equiv (B_x, B_y, B_0)\,,\ \vec{J} \equiv (J_x, J_y, 0)\,,\ \vec{E} \equiv (E_x, E_y, E_z)\,, \tag{8}$$

which are in agreement with the fundamental equations of Magnetohydrodynamics i.e. equations (1) to (6).

It is assumed that the induced magnetic field produced by motion of fluid is negligible in comparison to the applied one so that $\vec{B} \equiv (0, 0, B_0)$. This assumption is valid because magnetic Reynolds number is very small for metallic liquids and partially ionized fluids (Cramer and Pai, 1973). Also no external electric field is applied so the effect of polarization of fluid is neglected (Meyer, 1958) i.e $\vec{E} \equiv (0, 0, 0)$.

Under the above assumptions, equation (1) with the help of (6) and equation (7) reduce to

$$\frac{\partial u'}{\partial t'}-2\Omega v'=-\frac{1}{\rho}\frac{\partial p'}{\partial x}+\upsilon\frac{\partial^2 u'}{\partial z^2}-\frac{\upsilon}{K'}u'-\frac{\sigma B_0{}^2}{\rho}u'+g\beta'\left(T'-T_0\right), \tag{9}$$

$$\frac{\partial v'}{\partial t'}+2\Omega u'=\upsilon\frac{\partial^2 v'}{\partial z^2}-\frac{\upsilon}{K'}v'-\frac{\sigma B_0{}^2}{\rho}v', \tag{10}$$

$$0=-\frac{1}{\rho}\frac{\partial p'}{\partial z}, \tag{11}$$

$$\frac{\partial T'}{\partial t'}=\frac{k}{\rho c_p}\frac{\partial^2 T'}{\partial z^2}-\frac{Q_0}{\rho c_p}\left(T'-T_0\right), \tag{12}$$

We have considered oscillatory Hartmann convective flow so pressure p' is assumed in the following form

$$p'=2Rx\cos(\omega' t')+F(y)+G(z)\,. \tag{13}$$

It is noticed from equations (9), (10), (11) and (13) that pressure p' is constant along the axis of rotation i.e. $\frac{\partial p'}{\partial z}=G'(z)=0$. The absence of pressure gradient term $\frac{\partial p'}{\partial y}=F'(y)$ in equation (10) implies that there is a net cross flow in y-direction (Prasad Rao *et al*, 1982). Buoyancy term $g\beta'(T-T_0)$ is considered in equation (1) only because free-convection in this problem takes place under gravitational force (Singh, 1983; Tokis, 1986, 1988; Kythe and Puri, 1988 a, b; Nanousis, 1992 and Jha and Ajibade, 2009).
Boundary conditions for the fluid velocity are hydrodynamic slip boundary conditions which are given by

$$\left.\begin{aligned}&\mu\frac{\partial u'}{\partial z}=-\beta u' \text{ and } \mu\frac{\partial v'}{\partial z}=-\beta v' \quad \text{at } z=0,\\ &\mu\frac{\partial u'}{\partial z}=\beta u' \text{ and } \mu\frac{\partial v'}{\partial z}=\beta v' \quad \text{at } z=L.\end{aligned}\right\} \tag{14}$$

Boundary conditions (14) for the fluid velocity are well known hydrodynamic slip boundary conditions derived by Beavers and Joseph (1967). Here μ and β are, respectively, coefficient of dynamic viscosity and coefficient of sliding friction.
Boundary conditions for the fluid temperature are

$$\left.\begin{aligned}&T'=T_0 \text{ at } z=0,\\ &T'=T_0+\left(T_w-T_0\right)\cos\omega' t' \text{ at } z=L,\end{aligned}\right\} \tag{15}$$

where $T_0<T'<T_w$.
Equations (9), (10) and (12), in non-dimensional form, become

$$\frac{\partial u}{\partial t}-2K^2 v=-\frac{\partial p}{\partial \zeta}+\frac{\partial^2 u}{\partial \eta^2}-\frac{1}{K_1}u-M^2 u+G_r T, \tag{16}$$

$$\frac{\partial v}{\partial t}+2K^2 u=\frac{\partial^2 v}{\partial \eta^2}-\frac{1}{K_1}v-M^2 v, \tag{17}$$

$$\frac{\partial T}{\partial t}=\frac{1}{P_r}\frac{\partial^2 T}{\partial \eta^2}-\phi T, \tag{18}$$

where

$\zeta=x/L$, $\eta=z/L$, $u=u'L/\upsilon$, $v=v'L/\upsilon$, $t=t'\upsilon/L^2$, $p=L^2 p'/\rho\upsilon^2$, $T=\left(T'-T_0\right)/\left(T_w-T_0\right)$,
$K^2=\Omega L^2/\upsilon$, $K_1=K'/L^2$, $M^2=B_0{}^2L^2\left(\sigma/\rho\upsilon\right)$, $P_r=\upsilon\rho c_p/k$, $G_r=g\beta'\left(T_w-T_0\right)L^3/\upsilon^2$ *and* $\phi=Q_0L^2/\upsilon\rho c_p$.

K^2, M^2, G_r, P_r, K_1 and ϕ are rotation parameter which is reciprocal of Ekman number, magnetic parameter which is square of Hartmann number, Grashof number, Prandtl number, permeability parameter and heat generation/absorption coefficient respectively.
Boundary conditions (14) and (15), in dimensionless form, are

$$\left.\begin{aligned}&u=-\alpha\frac{\partial u}{\partial \eta} \text{ and } v=-\alpha\frac{\partial v}{\partial \eta} \text{ at } \eta=0,\\ &u=\alpha\frac{\partial u}{\partial \eta} \text{ and } v=\alpha\frac{\partial v}{\partial \eta} \text{ at } \eta=1,\end{aligned}\right\} \tag{19}$$

$$T=0 \text{ at } \eta=0 \text{ and } T=\cos\omega t \text{ at } \eta=1, \tag{20}$$

where $\alpha = \mu / \beta L$ is slip parameter and $\omega = \omega' L^2 / \upsilon$ is frequency parameter.
Equations (16) and (17), in compact form, become

$$\frac{\partial F}{\partial t} + 2iK^2 F = -\frac{\partial p}{\partial \zeta} + \frac{\partial^2 F}{\partial \eta^2} - \frac{1}{K_1} F - M^2 F + G_r T, \tag{21}$$

where $F = u + iv$.
Boundary conditions (19), in compact form, are

$$F + \alpha \frac{\partial F}{\partial \eta} = 0 \text{ at } \eta = 0 \text{ and } F - \alpha \frac{\partial F}{\partial \eta} = 0 \text{ at } \eta = 1. \tag{22}$$

It may be noted that the fluid flow past a plate may be induced due to either by motion of the plate or free stream or by heating of the fluid or by both (Singh, 1983; Tokis, 1986, 1988; Kythe and Puri, 1988 a, b; Kim, 2000; Chamkha, 2004 and Mbeledogu and Ogulu, 2007). Convective fluid flow within the channel may be induced due to either by heating of the fluid (Jha and Ajibade, 2009) or by the movement of one of the plates of the channel and heating of the fluid (Singh and Kumar, 2009) or by applied pressure gradient and heating of the fluid (Prasad Rao *et al*, 1982; Ghosh and Bhattacharjee, 2000; Seth and Singh, 2008 and Seth and Ansari, 2009). We have considered oscillatory Hartmann convective flow so fluid flow, in our case, is induced due to applied oscillatory pressure gradient and by heating of the fluid because of temperature difference between lower and upper plates. Therefore, pressure gradient $\frac{\partial p}{\partial \varsigma}$, fluid velocity $F(\eta, t)$ and fluid temperature $T(\eta, t)$ are assumed, in non-dimensional form, as

$$\frac{\partial p}{\partial \varsigma} = R\left(e^{i\omega t} + e^{-i\omega t}\right), \tag{23}$$

$$F(\eta, t) = F_1(\eta) e^{i\omega t} + F_2(\eta) e^{-i\omega t}, \tag{24a}$$

$$T(\eta, t) = T_1(\eta) e^{i\omega t} + T_2(\eta) e^{-i\omega t}, \tag{24b}$$

where $R<0$ for favourable pressure.
Equations (18) and (21) with the use of (23) and (24) reduce to

$$\frac{d^2 T_1}{d\eta^2} - P_r\left(\phi + i\omega\right) T_1 = 0, \tag{25}$$

$$\frac{d^2 T_2}{d\eta^2} - P_r\left(\phi - i\omega\right) T_2 = 0, \tag{26}$$

$$\frac{d^2 F_1}{d\eta^2} - \left\{\frac{1}{K_1} + M^2 + i\left(2K^2 + \omega\right)\right\} F_1 = R - G_r T_1, \tag{27}$$

$$\frac{d^2 F_2}{d\eta^2} - \left\{\frac{1}{K_1} + M^2 + i\left(2K^2 - \omega\right)\right\} F_2 = R - G_r T_2. \tag{28}$$

Boundary conditions (20) and (22) become

$$\left.\begin{aligned} &T_1 = 0 \text{ and } T_2 = 0 \text{ at } \eta = 0, \\ &T_1 = 1/2 \text{ and } T_2 = 1/2 \text{ at } \eta = 1, \end{aligned}\right\} \tag{29}$$

$$\left.\begin{aligned} &F_1 + \alpha \frac{dF_1}{d\eta} = 0 \text{ and } F_2 + \alpha \frac{dF_2}{d\eta} = 0 \text{ at } \eta = 0, \\ &F_1 - \alpha \frac{dF_1}{d\eta} = 0 \text{ and } F_2 - \alpha \frac{dF_2}{d\eta} = 0 \text{ at } \eta = 1. \end{aligned}\right\} \tag{30}$$

Equations (25) to (28) subject to boundary conditions (29) and (30) are solved and the solution for fluid temperature and fluid velocity is presented in the following form

$$T\left(\eta, t\right) = \frac{1}{2}\left[\frac{\sinh m_1 \eta}{sinh m_1} e^{i\omega t} + \frac{\sinh m_3 \eta}{sinh m_3} e^{-i\omega t}\right], \tag{31}$$

$$F\left(\eta, t\right) = \left\{C_1 \cosh m_2 \eta + C_2 \sinh m_2 \eta - \frac{R}{m_2^{\,2}} - \frac{G_r \sinh m_1 \eta}{2\left(m_1^{\,2} - m_2^{\,2}\right)\sinh m_1}\right\} e^{i\omega t} +$$

$$+\left\{C_3\cosh m_4\eta+C_4\sinh m_4\eta-\frac{R}{m_4^{\,2}}-\frac{G_r\sinh m_3\eta}{2\left(m_3^{\,2}-m_4^{\,2}\right)\sinh m_3}\right\}e^{-i\omega t}, \tag{32}$$

where

$$m_1=\left[P_r\left(\phi+i\omega\right)\right]^{1/2},\quad m_2=\left[\frac{1}{K_1}+M^2+i\left(2K^2+\omega\right)\right]^{1/2},$$

$$m_3=\left[P_r\left(\phi-i\omega\right)\right]^{1/2},\quad m_4=\left[\frac{1}{K_1}+M^2+i\left(2K^2-\omega\right)\right]^{1/2},$$

$$m_5=\left[\left(1+\alpha^2 m_2^2\right)\sinh m_2-2\alpha m_2\cosh m_2\right]^{-1},$$

$$m_6=\left[\left(1+\alpha^2 m_4^{\,2}\right)\sinh m_4-2\alpha m_4\cosh m_4\right]^{-1},$$

$$C_1=-m_5\left[\frac{R}{m_2}\left\{\alpha\left(1+\cosh m_2\right)-\frac{1}{m_2}\sinh m_2\right\}+\frac{\alpha m_2 G_r}{2\left(m_1^{\,2}-m_2^{\,2}\right)}\left\{1-\frac{\alpha m_1}{\sinh m_1}\left(\frac{\sinh m_2}{\alpha m_2}+\cosh m_1-\cosh m_2\right)\right\}\right],$$

$$C_2=m_5\left[\frac{G_r}{2\left(m_1^{\,2}-m_2^{\,2}\right)}\left\{1-\frac{\alpha m_1}{\sinh m_1}\left(\cosh m_1+\cosh m_2-\alpha m_2\sinh m_2\right)\right\}+\frac{R}{m_2^{\,2}}\left(1-\cosh m_2+\alpha m_2\sinh m_2\right)\right],$$

$$C_3=-m_6\left[\frac{R}{m_4}\left\{\alpha\left(1+\cosh m_4\right)-\frac{1}{m_4}\sinh m_4\right\}+\frac{\alpha m_4 G_r}{2\left(m_3^{\,2}-m_4^{\,2}\right)}\left\{1-\frac{\alpha m_3}{\sinh m_3}\left(\frac{\sinh m_4}{\alpha m_4}+\cosh m_3-\cosh m_4\right)\right\}\right],$$

$$C_4=m_6\left[\frac{G_r}{2\left(m_3^{\,2}-m_4^{\,2}\right)}\left\{1-\frac{\alpha m_3}{\sinh m_3}\left(\cosh m_3+\cosh m_4-\alpha m_4\sinh m_4\right)\right\}+\frac{R}{m_4^{\,2}}\left(1-\cosh m_4+\alpha m_4\sinh m_4\right)\right],$$

3. Asymptotic Solution

In order to gain further insight into the flow pattern, asymptotic behavior of the solution (32) will be analyzed for large values of frequency parameter ω.
i.e. when ω>>1, $M^2 \sim O(1)$ and $K^2 \sim O(1)$

When ω is large, boundary layer type flow is expected. For the boundary layer flow near the plate $\eta=1$, introducing boundary layer coordinate $\xi=1-\eta$, the asymptotic solution for the fluid velocity is obtained from (32) and is presented in the following form

$$u=\lambda\left[-\sqrt{P_r}\left\{e^{-\alpha_1\xi}\sin\left(\omega t-\beta_1\xi\right)+e^{-\alpha_2\xi}\sin\left(\omega t-\beta_2\xi\right)\right\}+2e^{-\alpha_3\xi}\sin\left(\omega t-\beta_3\xi\right)\right]+\frac{2R}{\omega}\left(\cos\omega t+\pi/2\right), \tag{33}$$

$$v=\lambda\sqrt{P_r}\left[e^{-\alpha_1\xi}\cos\left(\omega t-\beta_1\xi\right)-e^{-\alpha_2\xi}\cos\left(\omega t-\beta_2\xi\right)\right], \tag{34}$$

where

$$\lambda=\frac{G_r}{2\omega\left(1-P_r\right)}, \tag{35}$$

$$\alpha_1=\sqrt{\frac{\omega}{2}}\left\{1+\frac{K^2}{\omega}+\frac{1+K_1M^2}{2\omega K_1}\right\},\quad \beta_1=\sqrt{\frac{\omega}{2}}\left\{1+\frac{K^2}{\omega}-\frac{1+K_1M^2}{2\omega K_1}\right\}, \tag{36}$$

$$\alpha_2=\sqrt{\frac{\omega}{2}}\left\{1-\frac{K^2}{\omega}+\frac{1+K_1M^2}{2\omega K_1}\right\},\ \beta_2=\sqrt{\frac{\omega}{2}}\left\{1-\frac{K^2}{\omega}-\frac{1+K_1M^2}{2\omega K_1}\right\}, \tag{37}$$

$$\alpha_3=\sqrt{\frac{\omega P_r}{2}}\left(1+\frac{\phi}{2\omega}\right),\quad \beta_3=\sqrt{\frac{\omega P_r}{2}}\left(1-\frac{\phi}{2\omega}\right). \tag{38}$$

The expressions (33) to (38) demonstrate the existence of triple boundary layers of thicknesses $O(\alpha_1^{-1})$, $O(\alpha_2^{-1})$ and $O(\alpha_3^{-1})$ near the plate $\eta=1$. Two of the boundary layers of thicknesses $O(\alpha_1^{-1})$ and $O(\alpha_2^{-1})$ may be identified as modified

Sokes-Ekman boundary layers and can be viewed as classical Stokes-Ekman boundary layers modified by magnetic field and porosity of medium. The third boundary layer of thickness $O(\alpha_3^{-1})$ may be recognized as modified Stokes boundary layer and can be viewed as classical Stokes boundary layer modified by source/sink effect. Similar types of boundary layers are formed near the plate $\eta = 0$ of the channel. Exponential terms in the expressions (33) and (34) damp out quickly as ξ increases. When $\xi > \alpha_2^{-1}$ i.e. outside the boundary layer region, Eqs. (33) and (34) reduce to

$$u \approx \frac{2R}{\omega}\cos(\omega t + \pi / 2),\ v \approx 0. \tag{39}$$

It is evident from (39) that, in the central core region, fluid flows in the primary flow direction only and oscillates with the same frequency ω as applied pressure gradient but has a phase lead of $\pi / 2$ over it.

4. Skin Friction and Nusselt Number

The expressions for the skin friction τ and Nusselt number Nu, which are measure of shear stress due to primary and secondary flows and rate of heat transfer at the plate $\eta = 1$ respectively, are presented in the following form.

$$\tau = \left(\tau_x + i\tau_y\right) = \left\{C_1 m_2 \sinh m_2 + C_2 m_2 \cosh m_2 - \frac{G_r m_1}{2\left(m_1^2 - m_2^2\right)}\coth m_1\right\}e^{i\omega t} +$$
$$+\left\{C_3 m_4 \sinh m_4 + C_4 m_4 \cosh m_4 - \frac{G_r m_3}{2\left(m_3^2 - m_4^2\right)}\coth m_3\right\}e^{-i\omega t}, \tag{40}$$

and

$$Nu = -\frac{1}{2}\left[m_1 \coth m_1 e^{i\omega t} + m_3 \coth m_3 e^{-i\omega t}\right]. \tag{41}$$

5. Results and Discussion

To study the effects of wall slip, magnetic field, rotation, thermal buoyancy force, porosity of medium, oscillations and thermal source/sink on the flow-field numerical values of both primary and secondary fluid velocities, computed from analytical solution reported in Section 2 by MATLAB software, are displayed graphically versus channel width variable η for various values of slip parameter α, magnetic parameter M^2, rotation parameter K^2, Grashof number G_r, permeability parameter K_1, frequency parameter ω, heat generation coefficient $\phi\ (< 0)$ and heat absorption coefficient $\phi\ (> 0)$ in Figs. 2 to 15 taking $P_r = 0.71$, $\omega t = \pi / 2$ and $R = -1$. It is evident from Figs. 2 to 5 that primary velocity u and secondary velocity v decrease on increasing either slip parameter α or magnetic parameter M^2 for both heat generating and absorbing fluids which implies that wall slip and magnetic field have tendency to retard fluid flow in the primary and secondary flow directions for both heat generating and absorbing fluids. Figures 6 and 7 show that, for both heat generating and absorbing fluids, primary velocity u decreases whereas secondary velocity v increases with the increase in rotation parameter K^2 which implies that, for both heat generating and absorbing fluids, rotation tends to retard fluid flow in the primary flow direction whereas it has reverse effect on the fluid flow in secondary flow direction. Figures 8 to 13 reveal that, for both heat generating and absorbing fluids, u and v increase on increasing either G_r or K_1 or ω which implies that, for both heat generating and absorbing fluids, thermal buoyancy force, porosity of medium and oscillations have tendency to accelerate fluid flow in both the primary and secondary flow directions. It is noticed from Figs. 14 and 15 that u and v increase on increasing $\phi\ (< 0)$ and decrease on increasing $\phi\ (> 0)$ which implies that thermal source accelerates fluid flow in both the primary and secondary flow directions whereas thermal sink has reverse effect on it.

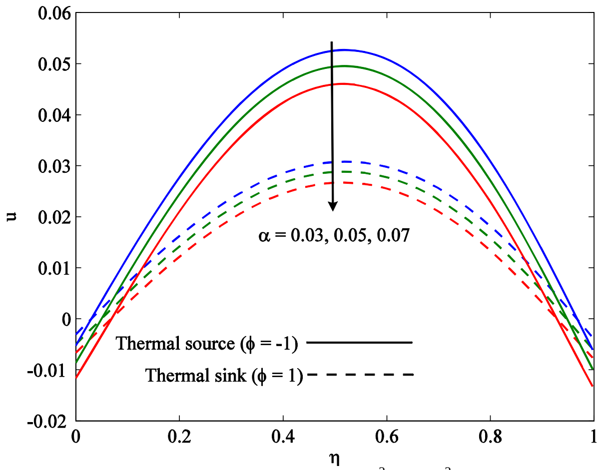

Figure 2. Profiles of primary velocity when M^2 = 4, K^2 = 3, G_r = 2, K_1=0.2 and $\omega = 3$.

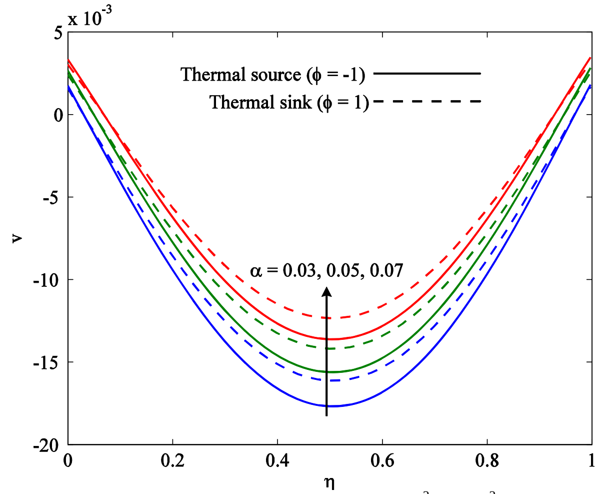

Figure 3. Profiles of secondary velocity when M^2 = 4, K^2 = 3, G_r = 2, K_1=0.2 and $\omega = 3$.

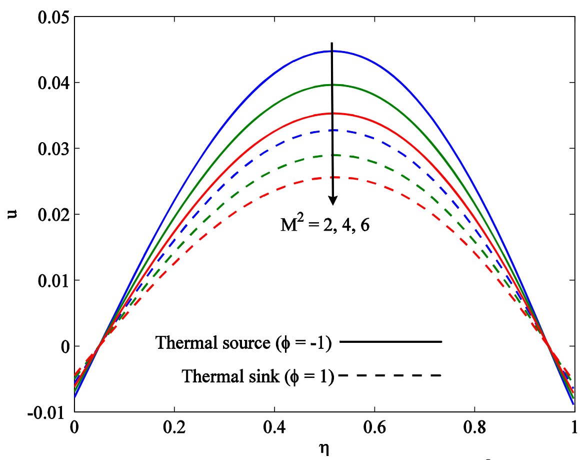

Figure 4. Profiles of primary velocity when $\alpha = 0.05$, K^2 = 3, G_r =2, K_1=0.2 and $\omega = 3$.

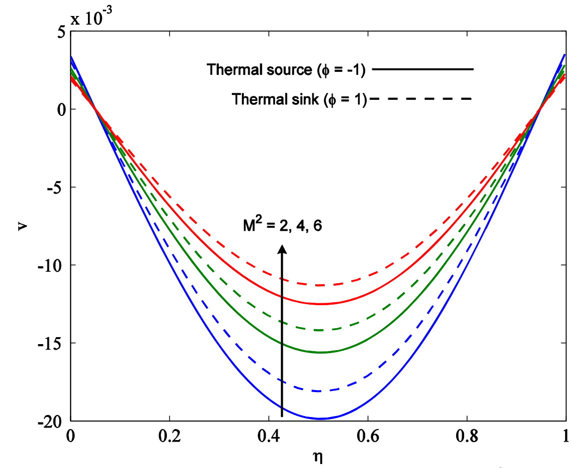

Figure 5. Profiles of secondary velocity when $\alpha = 0.05$, K^2 = 3, G_r=2, K_1=0.2 and $\omega = 3$.

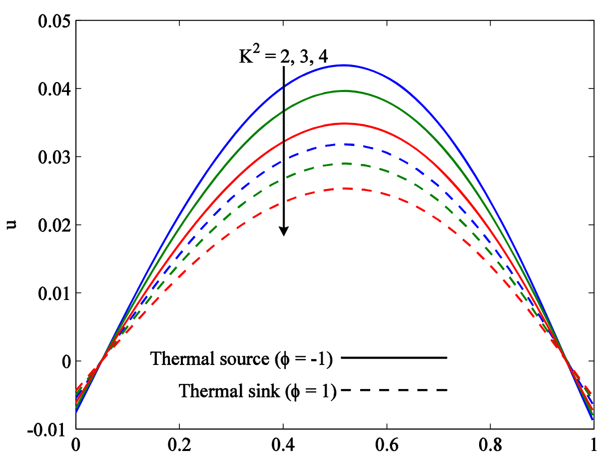

Figure 6. Profiles of primary velocity when $\alpha = 0.05$, M^2 = 4, G_r=2, K_1=0.2 and $\omega = 3$.

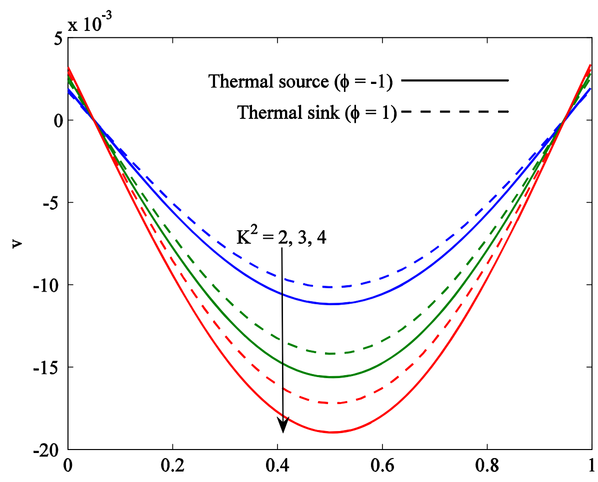

Figure 7. Profiles of secondary velocity when $\alpha = 0.05$, M^2 = 4, G_r=2, K_1=0.2 and $\omega = 3$.

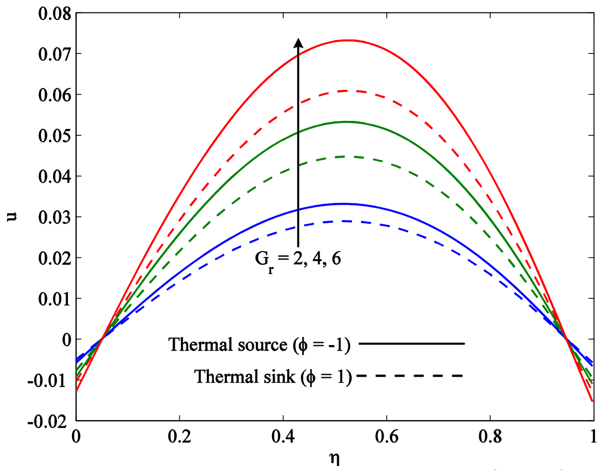

Figure 8. Profiles of primary velocity when $\alpha = 0.05$, $M^2 = 4$, $K^2 = 3$, $K_1 = 0.2$ and $\omega = 3$.

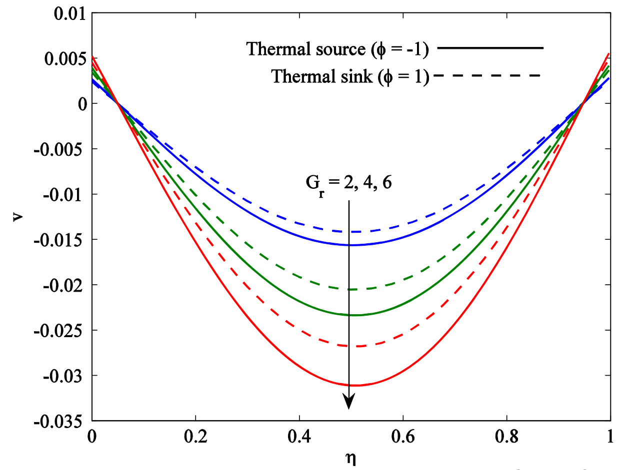

Figure 9. Profiles of secondary velocity when $\alpha = 0.05$, $M^2 = 4$, $K^2 = 3$, $K_1 = 0.2$ and $\omega = 3$.

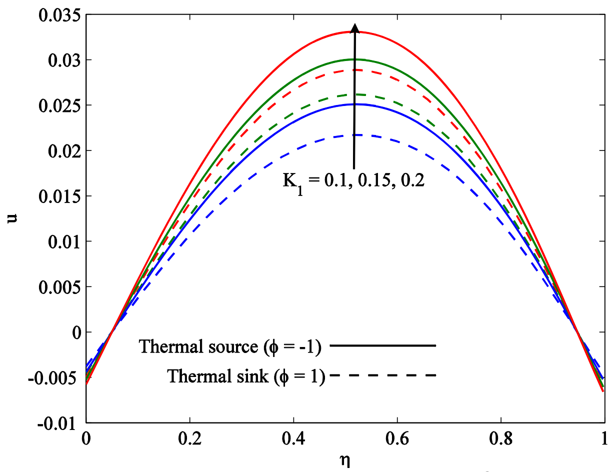

Figure 10. Profiles of primary velocity when $\alpha = 0.05$, $M^2 = 4$, $G_r = 2$, $K^2 = 3$ and $\omega = 3$.

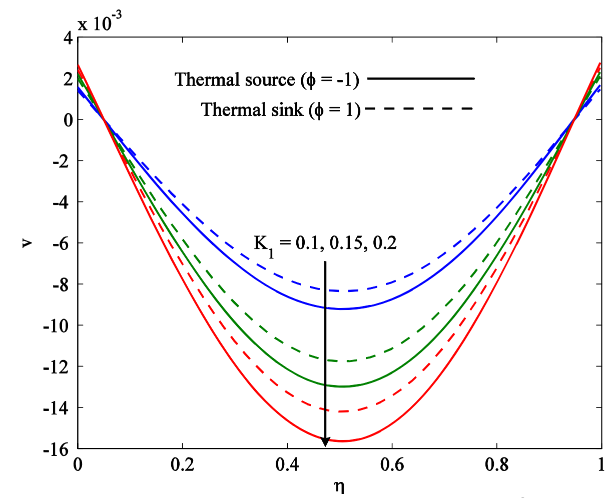

Figure 11. Profiles of secondary velocity when $\alpha = 0.05$, $M^2 = 4$, $G_r = 2$, $K^2 = 3$ and $\omega = 3$

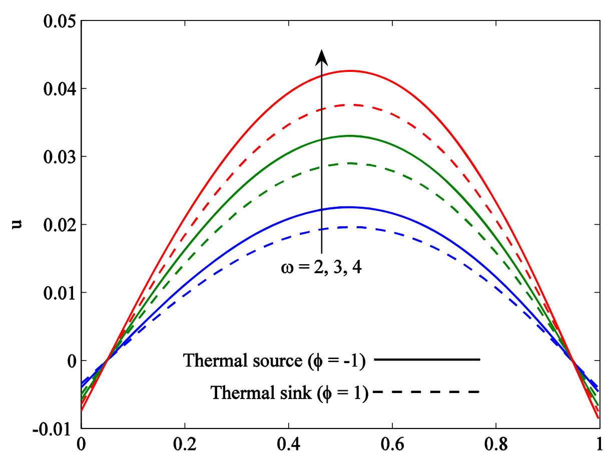

Figure 12. Profiles of primary velocity when $\alpha = 0.05$, $M^2 = 4$, $G_r = 2$, $K^2 = 3$ and $K_1 = 0.2$.

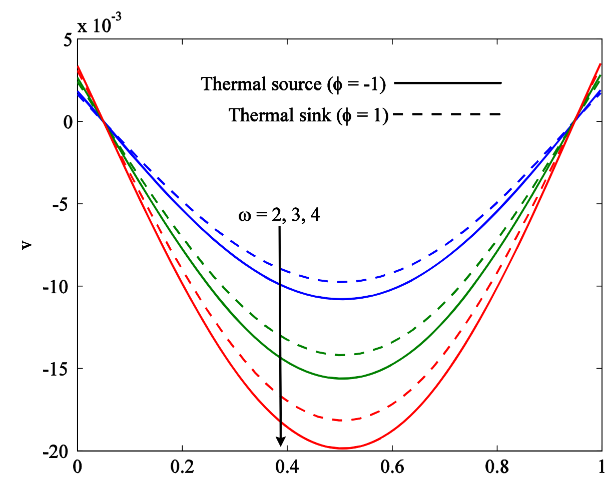

Figure 13. Profiles of secondary velocity when $\alpha = 0.05$, $M^2 = 4$, $G_r = 2$, $K^2 = 3$ and $K_1 = 0.2$.

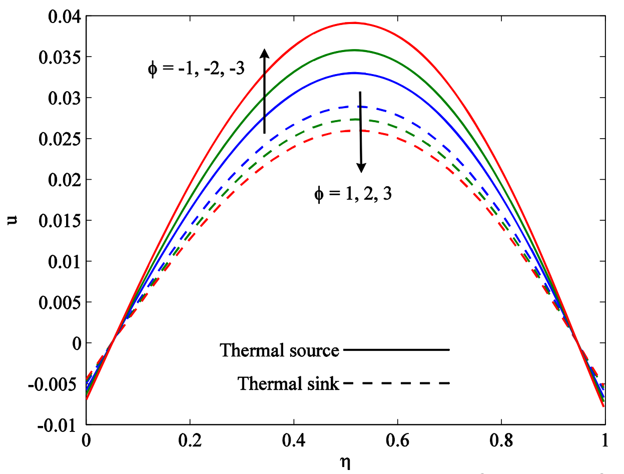

Figure 14. Profiles of primary velocity when $\alpha = 0.05$, $M^2 = 4$, G_r=2, K^2=3, $\omega = 3$ and $K_1 = 0.2$.

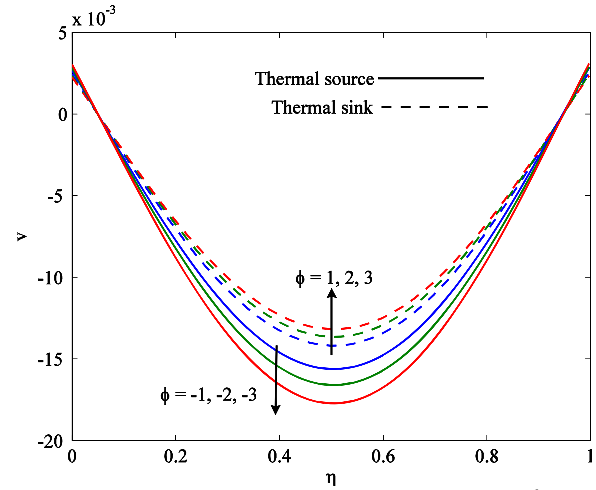

Figure 15. Profiles of secondary velocity when $\alpha = 0.05$, M^2 =4, G_r=2, K^2 = 3, $\omega = 3$ and $K_1 = 0.2$.

The numerical values of fluid temperature, computed from analytical solution mentioned in Section 2 by MATLAB software, are depicted graphically in Figs. 16 to 18 for different values of heat generation coefficient $\phi\ (<0)$, heat absorption coefficient $\phi\ (>0)$, Prandtl number P_r and frequency parameter ω taking $\omega t = \pi/2$. Figure 16 reveals that fluid temperature T increases on increasing $\phi\ (<0)$ and decreases on increasing $\phi\ (>0)$ which imply that thermal source tends to increase fluid temperature whereas thermal sink has reverse effect on it. Figure 17 shows that, for both heat generating and absorbing fluids, fluid temperature T increases on increasing Prandtl number P_r. Since Prandtl number P_r is ratio of viscosity to thermal diffusivity. An increase in thermal diffusivity leads to a decrease in Prandtl number. Therefore, thermal diffusion has tendency to reduce fluid temperature for both heat generating/absorbing fluids. It is noticed from Fig. 18 that, for both heat generating/absorbing fluids, fluid temperature T decreases in the lower half of the channel whereas it decreases, attains a minimum and then increases in magnitude in the upper half of the channel on increasing ω which implies that there exists reverse flow of heat in the upper half of the channel due to oscillating temperature of plate $\eta = 1$.

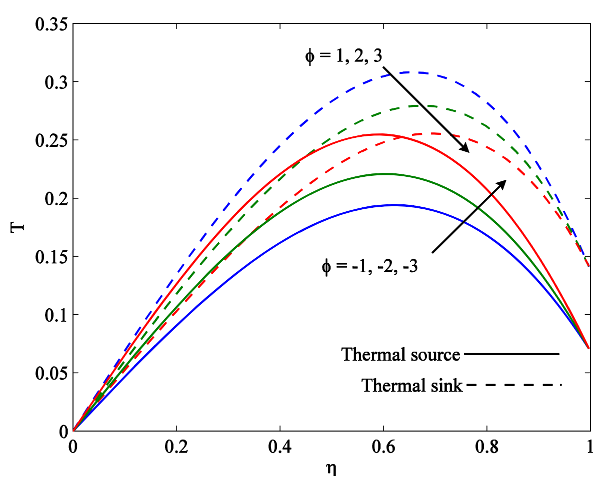

Figure 16. Temperature profiles when P_r = 0.71 and $\omega = 3$.

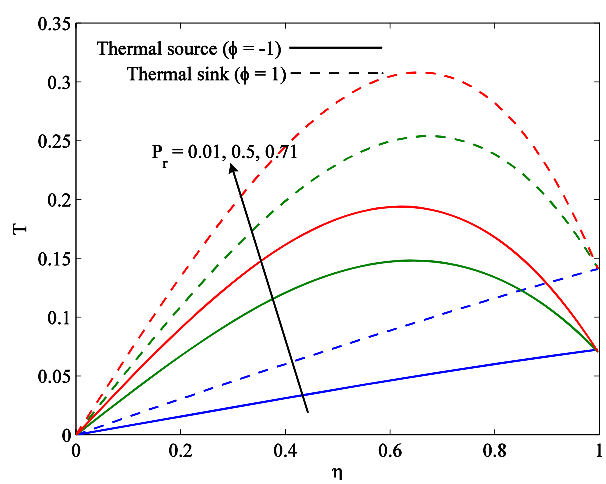

Figure 17. Temperature profiles when $\omega = 3$.

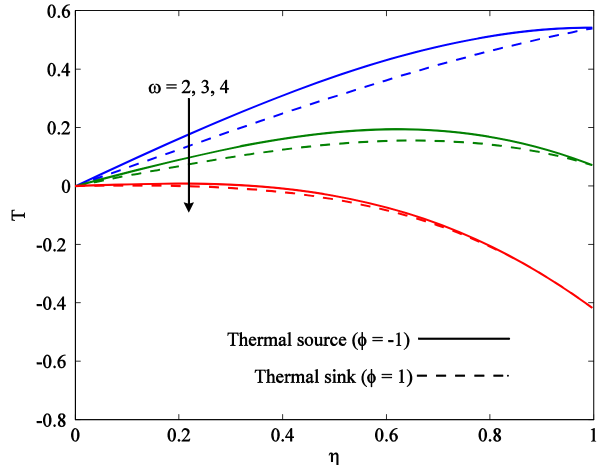

Figure 18. Temperature profiles when P_r = 0.71.

The numerical values of primary and secondary skin frictions at plate $\eta = 1$, computed from analytical expression reported in Section 4 by MATLAB software, are displayed in tabular form in Tables 1 to 8 for various values of M^2, G_r, K^2, ϕ, ω, K_1 and α taking $P_r = 0.71$, $\omega t = \pi/2$ and $R = -1$. It is evident from Tables 1 and 2 that, for both heat generating and absorbing fluids, primary skin friction τ_x and secondary skin friction τ_y decrease on increasing M^2 whereas these skin frictions increase on increasing G_r which implies that, for both heat generating and absorbing fluids, magnetic field tends to reduce both primary and secondary skin frictions whereas thermal buoyancy force has reverse effect on it. It is noticed from Tables 3 and 4 that, for both heat generating and absorbing fluids, τ_x decreases whereas τ_y increases on increasing K^2 which implies that, for both heat generating and absorbing fluids, rotation tends to reduce primary skin friction and it has reverse effect on secondary skin friction. τ_x and τ_y increase on increasing $\phi\ (<0)$ and decrease on increasing $\phi\ (>0)$ which implies that thermal source has tendency to increase both primary and secondary skin frictions whereas thermal sink has reverse effect on it. It is found from Tables 5 and 6 that, for both heat generating and absorbing fluids, τ_x and τ_y increase on increasing either ω or K_1 which implies that oscillations and porosity of medium tend to increase primary and secondary skin frictions for both heat generating and absorbing fluids. It is revealed from Tables 7 and 8 that, for both heat generating and absorbing fluids, τ_x and τ_y decrease on increasing α which implies that, for both heat generating and absorbing fluids, wall slip tends to reduce primary and secondary skin friction.

Table 1. Skin frictions τ_x and τ_y when $\alpha = 0.05$, $K^2 = 3$, $\omega = 3$, $K_1 = 0.2$ and $\phi = -1$

$M^2 \downarrow / G_r \rightarrow$	$-\tau_x$			τ_y		
	2	4	6	2	4	6
2	0.1473	0.2424	0.3374	0.0700	0.1046	0.1392
4	0.1321	0.2191	0.3061	0.0553	0.0832	0.1111
6	0.1192	0.1994	0.2795	0.0444	0.0674	0.0903

Table 2. Skin frictions τ_x and τ_y when $\alpha = 0.05$, $K^2 = 3$, $\omega = 3$, $K_1 = 0.2$ and $\phi = 1$

$M^2 \downarrow / G_r \rightarrow$	$-\tau_x$			τ_y		
	2	4	6	2	4	6
2	0.1293	0.2064	0.2835	0.0639	0.0924	0.1209
4	0.1156	0.1860	0.2565	0.0503	0.0733	0.0962
6	0.1040	0.1688	0.2337	0.0403	0.0592	0.0780

Table 3. Skin frictions τ_x and τ_y when $\alpha = 0.05$, $M^2 = 4$, $G_r = 2$, $K_1 = 0.2$ and $\omega = 3$.

$\phi \downarrow / K^2 \rightarrow$	$-\tau_x$			τ_y		
	2	3	4	2	3	4
-1	0.1432	0.1321	0.1186	0.0394	0.0553	0.0673
-2	0.1547	0.1431	0.1289	0.0417	0.0585	0.0713
-3	0.1690	0.1566	0.1416	0.0445	0.0624	0.0762

Table 4. Skin frictions τ_x and τ_y when $\alpha = 0.05$, $M^2 = 4$, $G_r = 2$, $K_1 = 0.2$ and $\omega = 3$.

$\phi \downarrow / K^2 \rightarrow$	$-\tau_x$			τ_y		
	2	3	4	2	3	4
1	0.1258	0.1156	0.1031	0.0360	0.0503	0.0611
2	0.1192	0.1092	0.0972	0.0346	0.0484	0.0587
3	0.1135	0.1039	0.0922	0.0334	0.0467	0.0567

Table 5. Skin frictions τ_x and τ_y when $\alpha = 0.05$, $M^2 = 4$, $G_r = 2$, $K^2 = 3$ and $\phi = -1$.

$\omega \downarrow / K_1 \rightarrow$	$-\tau_x$			τ_y		
	0.10	0.15	0.20	0.10	0.15	0.20
2	0.0706	0.0827	0.0901	0.0227	0.0317	0.0381
3	0.1035	0.1212	0.1321	0.0330	0.0460	0.0553
4	0.1340	0.1567	0.1707	0.0423	0.0587	0.0703

Table 6. Skin frictions τ_x and τ_y when $\alpha = 0.05$, $M^2 = 4$, $G_r = 2$, $K^2 = 3$ and $\phi = 1$.

$\omega \downarrow / K_1 \rightarrow$	$-\tau_x$			τ_y		
	0.10	0.15	0.20	0.10	0.15	0.20
2	0.0609	0.0717	0.0784	0.0204	0.0286	0.0345
3	0.0898	0.1058	0.1156	0.0298	0.0418	0.0503
4	0.1170	0.1377	0.1505	0.0385	0.0537	0.0644

Table 7. Skin frictions τ_x and τ_y when $K_1 = 0.2$, $M^2 = 4$, $G_r = 2$, $K^2 = 3$ and $\phi = -1$.

$\omega \downarrow / \alpha \rightarrow$	$-\tau_x$			τ_y		
	0.03	0.05	0.07	0.03	0.05	0.07
2	0.0923	0.0901	0.0855	0.0419	0.0381	0.0341
3	0.1354	0.1321	0.1252	0.0607	0.0553	0.0494
4	0.1753	0.1707	0.1615	0.0772	0.0703	0.0630

Table 8. Skin frictions τ_x and τ_y when $K_1 = 0.2$, $M^2 = 4$, $G_r = 2$, $K^2 = 3$ and $\phi = 1$.

$\omega \downarrow / \alpha \rightarrow$	$-\tau_x$			τ_y		
	0.03	0.05	0.07	0.03	0.05	0.07
2	0.0812	0.0784	0.0732	0.0381	0.0345	0.0308
3	0.1197	0.1156	0.1078	0.0555	0.0503	0.0449
4	0.1561	0.1505	0.1401	0.0710	0.0644	0.0576

Numerical values of Nusselt number Nu, computed from analytical expression mentioned in Section 4 by MATLAB software, are presented in tabular form in Tables 9 and 10 for different values of P_r, ω and ϕ taking $\omega t = \pi / 2$. It is found from Tables 9 and 10 that, for both heat generating and absorbing fluids, Nu increases on increasing either P_r or ω which implies that thermal diffusion tends to reduce rate of heat transfer at plate $\eta = 1$ whereas oscillations have reverse effect on it for both heat generating and absorbing fluids. Also Nu increases on increasing $\phi\ (< 0)$ and decreases on increasing $\phi\ (> 0)$ which implies that thermal source has tendency to enhance rate of heat transfer at plate $\eta = 1$ whereas thermal sink has reverse effect on it.

Table 9. Nusselt number Nu when $\omega = 3$.

$P_r \downarrow / \phi \rightarrow$	1	2	3	-1	-2	-3
0.01	0.0100	0.0100	0.0100	0.0100	0.0100	0.0100
0.5	0.4631	0.4369	0.4138	0.5273	0.5670	0.6134
0.71	0.6341	0.5868	0.5468	0.7587	0.8424	0.9469

Table 10. Nusselt number Nu when $P_r = 0.71$.

$\omega \downarrow / \phi \rightarrow$	1	2	3	-1	-2	-3
2	0.4281	0.3955	0.3679	0.5152	0.5744	0.6491
3	0.6341	0.5868	0.5468	0.7587	0.8424	0.9469
4	0.8309	0.7711	0.7200	0.9868	1.0900	1.2169

6. Conclusion

Unsteady hydromagnetic convective flow of a viscous incompressible electrically conducting heat generating/absorbing fluid within a parallel plate rotating channel in a porous medium under slip boundary conditions is investigated. The significant findings are summarized below:

a). For both heat generating and absorbing fluids:
  (i). wall slip and magnetic field have tendency to retard fluid flow in both the primary and secondary flow directions.
  (ii). rotation tends to retard fluid flow in primary flow direction whereas it has reverse effect on fluid flow in secondary flow direction.
  (iii). buoyancy force, porosity of medium and oscillations have tendency to accelerate fluid flow in both the primary and secondary flow direction.

b). Thermal source tends to accelerate fluid flow in both the primary and secondary flow directions whereas thermal sink has reverse effect on it.

c). (i). Thermal source tends to enhance fluid temperature whereas thermal sink has reverse effect on it.
  (ii).Thermal diffusion has tendency to reduce fluid temperature for both heat generating and absorbing fluids.
  (iii).Oscillations tend to induce reverse flow of heat for both heat generating and absorbing fluids in the upper half of the channel due to oscillating temperature of upper plate.

d). For both heat generating and absorbing fluids:
  (i). magnetic field tends to reduce both primary and secondary skin frictions whereas thermal buoyancy force has reverse effect on it.
  (ii). rotation tends to reduce primary skin friction whereas it has reverse effect on secondary skin friction.
  (iii). oscillations and porosity of medium tend to increase both primary and secondary skin frictions whereas wall slip has reverse effect on it.

e). Thermal source has tendency to increase both primary and secondary skin frictions whereas thermal skink has reverse effect on it.

f). Thermal source has tendency to enhance rate of heat transfer at plate $\eta = 1$ whereas thermal sink has reverse effect on it. Thermal diffusion tends to reduce rate of heat transfer at plate $\eta = 1$ whereas oscillations have reverse effect on it for both heat generating and absorbing fluids.

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